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Sheet (3)

(1) $i_2 = I_m \cos(\omega t + \beta)$ $\quad i_1 = 0$

$M = M_{max} \cos(\omega t)$ $\quad \theta = 0$ at $t = 0$

$\lambda = 1.0$ wb at $t = 0$ $\quad \omega_m = 3000$ rpm $\times \frac{2\pi}{60} = 314$ rad/s

$\lambda_{11} = L_{11} \underbrace{i_1}_{0} + M\, i_2$

$\lambda_{11} = M_{max} \cos(\omega t)\, I_m \cos(\omega t)$

$1.0 = M_{max} I_{max} \longrightarrow (1)$

$V_{1t} = L_{11} \frac{di_1}{dt} + M \frac{di_2}{dt}$

$= M_{max} \cos(\omega t) \cdot -\omega I_m \sin(\omega t)$

$V_{1t} = -1570 \sin(628t)$

$V_{1m} = \omega_m \left[i_1 \underbrace{\frac{\partial L_{11}}{\partial \theta}}_{0} + i_2 \frac{\partial M}{\partial \theta} \right]$

$= \omega_m i_2 \cdot - M_{max} \sin(\omega t)$

$= -\omega_m I_m \cos(\omega t) \cdot M_{max} \sin(\omega t)$

$= -1570 \sin(628t)$

$V_1 = V_{1t} + V_{1m} = -3140 \sin(628t)$ V

$V_{1\,rms} = 2220$ V $\quad f = 100$ Hz

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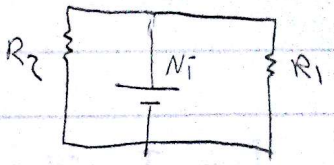
(2)



$$R_1 = \frac{l_1}{\mu_0 A_1} = \frac{2.52 \times 10^{-3}}{\mu_0 \times 2 \times 1 \times 10^{-4}}$$

$$= 1002b761 \text{ AT/wb}$$

$$R_2 = \frac{l_2}{\mu_0 A_2} = \frac{1.26 \times 10^{-3}}{\mu_0 \times 1 \times 10^{-4}} = 1002b761 \text{ AT/wb}$$

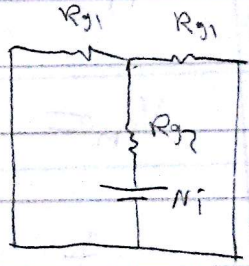
$$\phi_1 = \frac{Ni}{R_1} = \frac{1000 \times 1}{R_1} = 9.97 \times 10^{-5} \text{ wb}$$

$$\phi_2 = 9.97 \times 10^{-5} \text{ wb}$$

$$\lambda = N\phi_t = 1000 \times 2 \times 9.97 \times 10^{-5} = 0.199 \text{ wb.T}$$

$$W = \frac{1}{2} \lambda I = \frac{1}{2} \times 0.199 \times 1 = 99.73 \text{ mJ}$$

(3)



$$R_{g1} = \frac{g}{\mu_0 w d}$$

$$R_{g2} = \frac{x}{\mu_0 \cdot 2 w d}$$

$$R_T = R_{g2} + R_{g1} // R_{g1} = \frac{g + x}{2 \mu_0 w d}$$

$$\phi_T = \frac{Ni}{R_T} = \frac{2Ni \mu_0 w d}{g + x}$$

$$L = \frac{N^2}{R_T} = \frac{2N^2 \mu_0 w d}{g + x}$$

$$W = \frac{1}{2} L i^2 = \frac{N^2 i^2 \mu_0 w d}{g + x}$$

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(4) at $B = 1.2\ T$ $H = 200\ AT/m$ (from BH curve)

$$\mu_c = \frac{B}{H} = 6 \times 10^{-3}$$

$$R_c = \frac{l_c}{\mu_c A_c} = \frac{(4+9+4+9-0.1)\times 10^{-2}}{6\times 10^{-3} \times 10^{-4}}$$

$$R_c = 431666.7\ AT/wb$$

$$R_g = \frac{l_g}{\mu_0 A_g} = 7957747.15\ AT/wb$$

$$L = \frac{N^2}{R} = 4.768\ mH$$

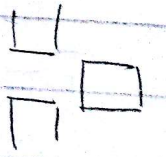
$$W = \frac{1}{2} L I^2 = \frac{1}{2}\frac{\lambda^2}{L} = \frac{(N\Phi)^2}{2L} = \boxed{\frac{(NBA)^2}{2L}} \rightarrow \text{Ans}$$

$$= \frac{(200 \times 1.2 \times 10^{-4})^2}{2 \times 4.768 \times 10^{-3}} = 60.4\ mJ$$

(5) $\mu_r = 10000$ $I = 1A$

$$W_m = \frac{1}{2} I(\lambda_2 - \lambda_1) = \frac{1}{2} I^2 (L_2 - L_1)$$

position 1:



$$R_T = \frac{31.4\times 10^{-2}}{10000\ \mu_0 \times 10^{-4}} + \frac{3.14\times 10^{-3}}{\mu_0 \times 10^{-4}}$$

$$R_T = 25237199\ AT/wb$$

$$L_1 = \frac{N^2}{R_T} = 99\ \mu H$$

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position 2:

$$R_2 = \frac{(31.4 + 0.3141) \times 10^{-2}}{10000 \times \mu_0 \times 10^{-7}}$$

$$R_2 = 252371.99 \text{ AT/wb}$$

$$L_2 = 9.9 \text{ mH}$$

$$W_m = \frac{1}{2} \times (1)^2 \times (9.9 \times 10^{-3} - 99 \times 10^{-6})$$

$$W_m = 4.9 \text{ mJ}$$

(b) assuming the system is linear:

$$W_{fld} = W'_{fld} = \frac{1}{2} L i^2 = \frac{1}{2} i^2 (L_1 + L_2 \cos(2\theta_m))$$

$$T = \frac{\partial W'_{fld}(i,\theta)}{\partial \theta} = \frac{1}{2} i^2 \cdot L_2 \cdot -2 \sin(2\theta_m)$$

$$\boxed{T = -L_2 i^2 \sin(2\theta_m)}$$

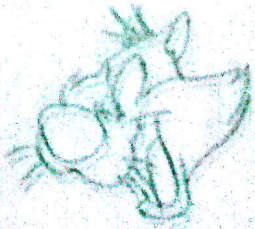
$$\theta_m = wt$$

$$T = -L_2 I_m^2 \cos^2(wt + \delta) \sin(2wt)$$

$$= -L_2 I_m^2 \left[\frac{1}{2} + \frac{1}{2}\cos(2wt + 2\delta)\right] \sin 2wt$$

$$T = -L_2 I_m^2 \left[\frac{1}{2}\sin 2wt + \frac{1}{4}(\sin(4wt + 2\delta) - \sin(2\delta))\right]$$

$$\boxed{T_{avr} = +\frac{1}{4} L_2 I_m^2 \sin(2\delta)}$$



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(7) $i = [A_0 + A_2 \cos(2\theta_m)]\, \lambda^2$

$$W_{fij} = \int_0^{\lambda_0} i \, d\lambda = \int_0^{\lambda_0} (A_0 + A_2 \cos 2\theta_m)\, \lambda^2 \, d\lambda$$

$$\boxed{W_{fij} = \frac{1}{3}(A_0 + A_2 \cos(2\theta_m))\, \lambda^3}$$

$$W_{fij} + W'_{fij} = i\lambda$$

$$W'_{fij} + \frac{1}{3}(A_0 + A_2 \cos 2\theta_m)\lambda^3 = (A_0 + A_2 \cos 2\theta_m)\lambda^3$$

$$\boxed{W'_{fij} = \frac{2}{3}[A_0 + A_2 \cos 2\theta_m]\lambda^3}$$

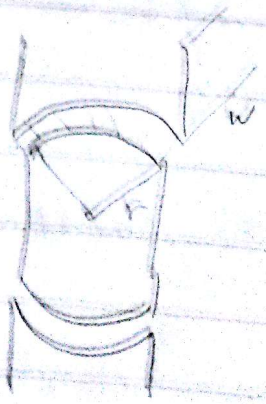
$$T = -\frac{\partial W_{fij}(\lambda,\theta)}{\partial \theta} = -\frac{1}{3}\lambda^3 \cdot -2A_2 \sin(2\theta_m)$$

$$\boxed{T = \frac{+2}{3}\lambda^3 A_2 \sin(2\theta_m)}$$

$I = \text{const}$

$$L = \frac{N^2}{R_T}$$

$$R_T = \frac{2\,g(\theta)}{}$$



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(9)

$$p = e_1 i_1 + e_2 i_2$$

$$e_1 = L_{11} \frac{di_1}{dt} + M \frac{di_2}{dt} + \dot{\theta} \left[i_1 \frac{\partial L_{11}}{\partial \theta} + i_2 \frac{\partial M}{\partial \theta} \right]$$

$$e_2 = L_{22} \frac{di_2}{dt} + M \frac{di_1}{dt} + \dot{\theta} \left[i_1 \frac{\partial M}{\partial \theta} + i_2 \frac{\partial L_{22}}{\partial \theta} \right]$$

(a) $$p_{elec} = i_1 L_{11} \frac{di_1}{dt} + M i_1 \frac{di_2}{dt} + L_{22} i_2 \frac{di_2}{dt} + M i_2 \frac{di_2}{dt} + \dot{\theta} \left[i_1^2 \frac{\partial L_{11}}{\partial \theta} + 2 i_1 i_2 \frac{\partial M}{\partial \theta} + i_2^2 \frac{\partial L_{22}}{\partial \theta} \right]$$

at constant i_1, i_2 :

$$p_{elec} = \dot{\theta} \left[i_1^2 \frac{\partial L_{11}}{\partial \theta} + i_2^2 \frac{\partial L_{22}}{\partial \theta} + 2 i_1 i_2 \frac{\partial M}{\partial \theta} \right]$$

$$W_{fld} = \frac{1}{2} L_{11} i_1^2 + \frac{1}{2} L_{22} i_2^2 + M i_1 i_2$$

(b) $$\partial W_{fld} = i_1 L_{11} \partial i_1 + \frac{1}{2} i_1^2 \partial L_{11} + L_{22} i_2 \partial i_2 + \frac{1}{2} i_2^2 \partial L_{22} + M i_1 \partial i_2 + M i_2 \partial i_1 + i_1 i_2 \partial M$$

at const currents

$$\partial W_{fld} = \frac{1}{2} i_1^2 \partial L_{11} + \frac{1}{2} i_2^2 \partial L_{22} + i_1 i_2 \partial M$$

$$\partial W_{elec} = \partial W_{fld} + \partial W_m$$

$$p_{mech} = p_{elec} - p_{fld}$$

$$= \dot{\theta} \left[\frac{1}{2} i_1^2 \frac{\partial L_{11}}{\partial \theta} + \frac{1}{2} i_2^2 \frac{\partial L_{22}}{\partial \theta} + i_1 i_2 \frac{\partial M}{\partial \theta} \right]$$

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10 $x+y=g \rightarrow$ const

$$R_T = \frac{x}{\mu_0 A} + \frac{y}{\mu_0 A} = \frac{g}{\mu_0 A}$$

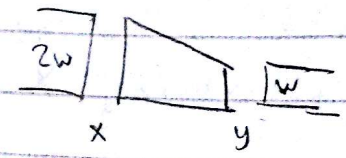
$$L = \frac{N^2}{R_T} = \frac{N^2 \mu_0 A}{g}$$

$$F = \frac{1}{2} i^2 \frac{\partial L(x)}{\partial x}_{x_0}$$

$$\boxed{F = 0}$$

11

$$R_T = \frac{x}{2\mu_0 w d} + \frac{y}{\mu_0 w d}$$



$$R_T = \frac{x+2y}{2\mu_0 w d}$$

$$L = \frac{2N^2 \mu_0 w d}{x+2y}$$

$$L(x) = \frac{2N^2 \mu_0 w d}{2g-x}$$

$x+y=g$

$y = g-x$

$x+2y = x+2g-2x$

$= 2g-x$

$$F = \frac{1}{2} I^2 \frac{\partial L(x)}{\partial x} = N^2 I^2 \mu_0 w d \frac{1}{(2g-x)^2}$$

$$F = \frac{N^2 I^2 \mu_0 w d}{(2g-x)^2}$$

in the first problem: inductance is not dependent on the the position as the air gap always has the same length and area. but in the second problem the air gap (and the inductance) depends on the position.

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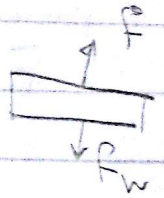
(12)

$$R_T = \frac{2x}{\mu_0 A}$$

$$L(x) = \frac{N^2 \mu_0 w d}{2x}$$

$$F = \frac{1}{4} I^2 N^2 \mu_0 w d \cdot \frac{d}{dx}\left(\frac{1}{x}\right)$$

$$F = -\frac{N^2 I^2 \mu_0 w d}{4x^2}$$



negative sign indicates that the force is in the direction of decreasing x

$$F = f_w$$

$$\frac{N^2 I^2 \mu_0 w d}{4x^2} = mg$$

$$I = \frac{2x}{N}\sqrt{\frac{mg}{\mu_0 w d}}$$

(13)

$$L(x) = \frac{N^2 \mu_0 w d}{2x}$$

$$W_{fld} = \frac{1}{2} L(x) I^2 = \frac{N^2 I^2 \mu_0 w d}{4x}$$

$$W_{fld} = \frac{(300 \times 20)^2 \times \mu_0 \times 6 \times 6 \times 10^{-4}}{4 \times 9 \times 10^{-3}} = 8.143 \text{ J}$$

$$F = \frac{-N^2 I^2 \mu_0 w d}{4x^2} = 1628.6 \text{ N}$$

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(14) missing: $\boxed{A = 4\,cm^2}$

$$R = \frac{x}{\mu_0 A}$$

$$L = \frac{N^2 \mu_0 A}{x}$$

$$F = \frac{1}{2} I^2 N^2 \mu_0 A \frac{d}{dx}\left(\frac{1}{x}\right)$$

$$F = -\frac{N^2 I^2 \mu_0 A}{2x^2}$$

$$W_{fld} = \frac{N^2 I^2 \mu_0 A}{2x}$$

a-

$$31.4 = \frac{1414\, I^2\, \mu_0 \times 4 \times 10^{-4}}{2 \times (10^{-3})^2}$$

$$\boxed{I = 9.4\ A}$$

$$W_{fld} = F x = 31.4 \times 10^{-3} = 31.4\ mJ$$

b-

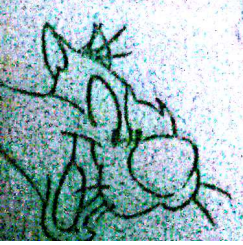
$$31.4 = \frac{1414\, \mu_0 \times 4 \times 10^{-4}\, I^2}{2 \times (2 \times 10^{-3})^2}$$

$$\boxed{I = 18.8\ A}$$

$$W_{fld} = 62.8\ mJ$$

→ expected result as $I \propto x$ when F is const

$$W_{fld} \propto \frac{I^2}{x}$$



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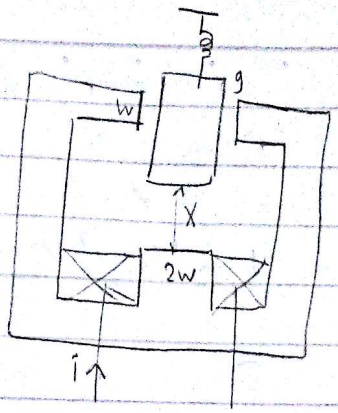
(15) $N = 200$ $\quad 2wd = 100\ cm^2 \rightarrow wd = 50\ cm^2$

$x = 0.5 \rightarrow 1\ cm$

$I = 2\ A$



$R_g = R_c = 0$

$$R_T = \frac{x}{\mu_0 2wd}$$

$$L = \frac{2N^2 \mu_0 wd}{x} \Rightarrow L(x) = \frac{502.65}{x}\ nH$$

$$W_{fld} = \frac{N^2 I^2 \mu_0 wd}{x} \Rightarrow W_{fld}(x) = \frac{1}{x}\ mJ$$

$$F = -\frac{N^2 I^2 \mu_0 wd}{x^2}$$

at $x = 1\ cm$

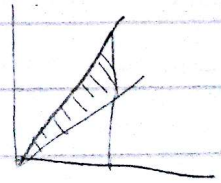
$$\boxed{F = 10\ N}$$

the external force is $10\ N$ vertically upward

at $x = 0.5$

$$\boxed{F = 40.21\ N}$$

move slowly → constant current operation

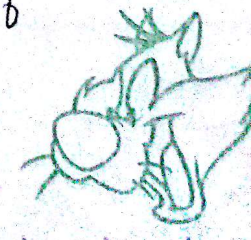


$$W_{mec} = \frac{1}{2} I (\lambda_2 - \lambda_1)$$

$$= \frac{1}{2} I^2 (L_2 - L_1)$$

$$= \frac{1}{2} (2)^2 \left(\frac{502.65}{0.5 \times 10^{-2}} - \frac{502.65}{10^{-2}} \right) \times 10^{-6}$$

$$\boxed{W_m = 0.1\ J}$$



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(16) $L_{11} = L_{22} = \frac{3.6}{1+2x} = 2M$ $\quad i_1 = i_2 = i \rightarrow$ (series)

$M = \frac{1.8}{1+2x}$

$e_1 = L_{11}\frac{di_1}{dt} + M\frac{di_2}{dt} = L\frac{di}{dt} + \frac{L}{2}\frac{di}{dt}$

$e_2 = M\frac{di_1}{dt} + L_{22}\frac{di_2}{dt} = \frac{L}{2}\frac{di}{dt} + L\frac{di}{dt}$

$V_s = e_1 + e_2$

$V_s = 3L\frac{di}{dt}$

at $x = 0.4$

$$100\cos(314t) = \frac{3 \times 3.6}{1+2\times0.4}\frac{di}{dt}$$

$$i(t) = 16.67\int \cos 314t\, dt$$

$$i(t) = 53.1 \sin(314t) \text{ mA}$$

$$W_{fld} = \frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{22}i_2^2 + M i_1 i_2$$

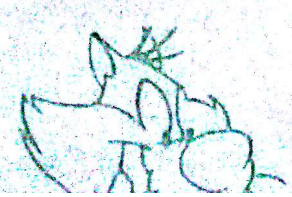
$$W_{fld} = \frac{3}{2}L i^2$$

$$F = \frac{3}{2} i^2 \frac{d}{dx}\left(\frac{3.6}{1+2x}\right)$$

$$= \frac{3}{8} i^2 \frac{-3.6\times8}{(1+2x)^2} = \frac{-10.8}{(1+2x)^2} i^2$$

at $x = 0.4$

$$F = 9.398\times10^{-3}\sin^2(314t) \text{ N}$$



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$$F = 9398.7 \left(\frac{1}{2} - \frac{1}{2}\cos(628t)\right)$$

$$F_{avr} = 4.699 \quad mN$$

---

(17) $g = 1\,mm \rightarrow 5\,mm \quad A = 5\,cm^2 \quad N = 1000\,turns$

$$R = \frac{g}{\mu_0 A}$$

$$L = \frac{N^2 \mu_0 A}{g}$$

$$W_{field} = \frac{N^2 \mu_0 A I^2}{2g}$$

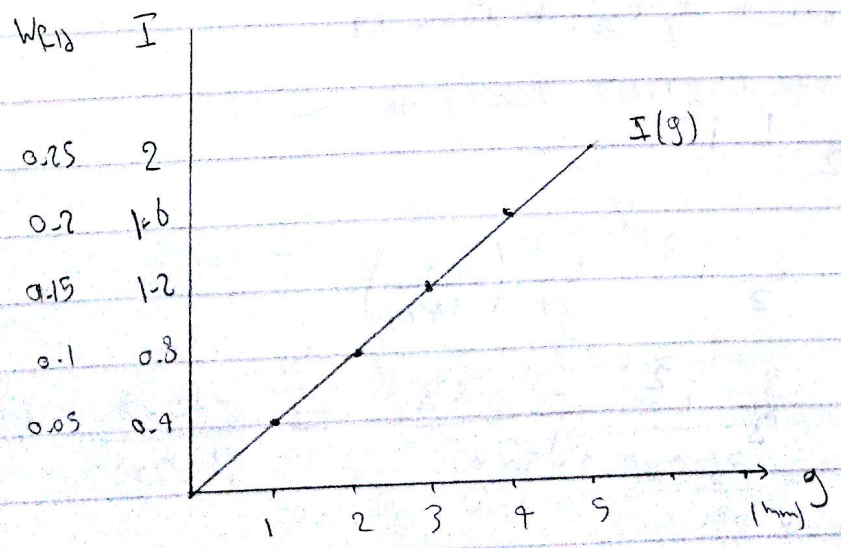
$$F = -\frac{N^2 I^2 \mu_0 A}{2g^2}$$

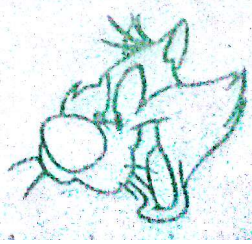
$$I^2 = \frac{2g^2 F}{N^2 \mu_0 A}$$

$$I = 398.94\, g \rightarrow (1)$$

$$W_{field} = 50\, g \rightarrow (2)$$

W_{field}	I
0.25	2
0.2	1.6
0.15	1.2
0.1	0.8
0.05	0.4





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(18)

$$W_{f1} = \int_0^{\lambda_0} i \, d\lambda$$

$$W'_{f1} = \int_0^{I_0} \lambda \, di = \int \frac{6i}{2i+1} \, di$$

$$= 3 \int \frac{2i+1 \; -1}{2i+1} \, di = 3 \int 1 - \frac{1/2}{2i + 1/2} \, di$$

$$= 3 \left[I_0 - \frac{1}{2} \ln \left(i + \frac{1}{2} \right) \Big|_0^{I_0} \right]$$

$$= 3 I_0 - \frac{3}{2} \left(\ln \left(I_0 + \frac{1}{2} \right) - \ln \frac{1}{2} \right)$$

when $\lambda_0 = 2$ wb

$$4i + 2 = 6i$$

$$I_0 = 1 \text{ A}$$

$$W'_{f1} = 1.352$$

$$W'_{f1} + W_{f1} = \lambda_0 I_0$$

$$W_{f1} = 2 - 1.352 = 0.648 \text{ J}$$

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(19) $L_{11} = (3 + \cos 2\theta) * 10^{-3}$ $\quad i_1 = 1$

$L_{22} = 30 + 10 \cos 2\theta$ $\quad i_2 = 0.01$

$M = 0.1 \cos\theta$

$$W_{fld} = \frac{1}{2} L_{11} i_1^2 + \frac{1}{2} L_{22} i_2^2 + M i_1 i_2$$

$$= \frac{1}{2} \times 10^{-3} (3 + \cos 2\theta) + 5 \times 10^{-5} (30 + 10 \cos 2\theta) + 10^{-3} \cos\theta$$

$$= 3 \times 10^{-3} + 10^{-3} \cos 2\theta + 10^{-3} \cos\theta$$

$$W_{fld} = (3 + \cos 2\theta + \cos\theta)\ mJ$$

$$T = \frac{\partial W_{fld}}{\partial \theta}\Big|_{I=\text{const}} = -2 \sin 2\theta - \sin\theta \quad mN\text{-}m$$

the position of maximum torque

$$-4 \cos 2\theta - \cos\theta = 0$$

$$4(2\cos^2\theta - 1) - \cos\theta = 0$$

$$8\cos^2\theta - \cos\theta - 4 = 0$$

$\cos\theta = -0.6473$ $\quad \theta = 130.34$

$\cos\theta = 0.77236$ $\quad \theta = 39.47^\circ$

Sketch

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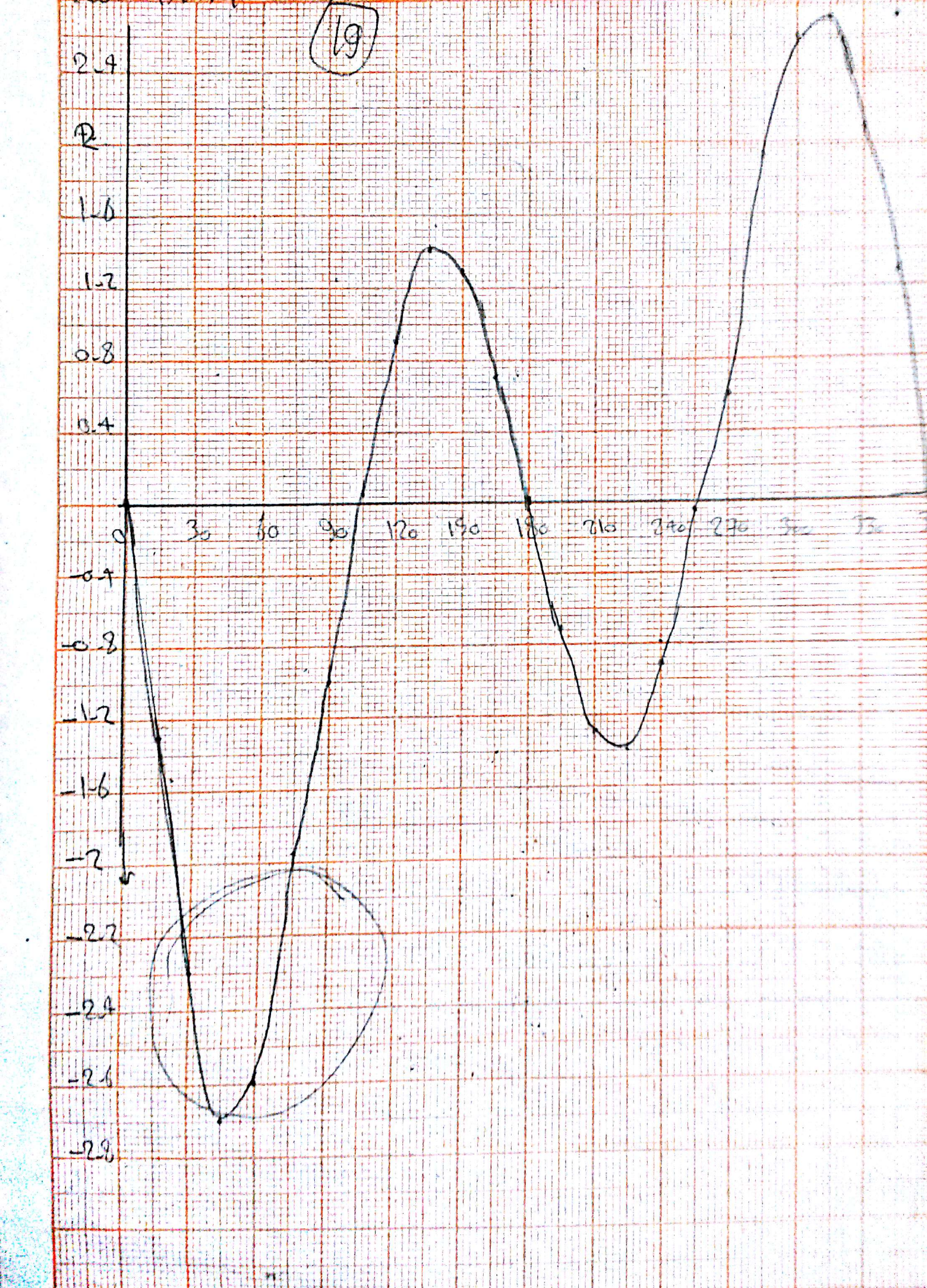

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(20) L_{11} = max 0.4 min 0.2 H $r_1 = 10\Omega$

L_{22} // 0.6 min 0.4 $r_2 = 20\Omega$

M = max 0.2

$L_{11} = 0.3 + 0.1 \cos 2\theta$ $i_1 = 1A$

$L_{22} = 0.5 + 0.1 \cos 2\theta$ $i_2 = 0.5A$

$M = 0.2 \cos\theta$

$$W_{fld} = \frac{1}{2} L_{11} i_1^2 + \frac{1}{2} L_{22} i_2^2 + M i_1 i_2$$

$$W_{fld} = 0.5 (0.3 + 0.1 \cos 2\theta) + \frac{1}{8} (0.5 + 0.1 \cos 2\theta) + 0.1 \cos\theta$$

$$W_{fld} = \frac{17}{80} + \frac{1}{16} \cos 2\theta + 0.1 \cos\theta$$

$$T = \frac{-1}{8} \sin 2\theta - 0.1 \sin\theta$$

---

(21) the same as ex 1.

$$\lambda_{11} = L_{11} i_1 + M i_2$$

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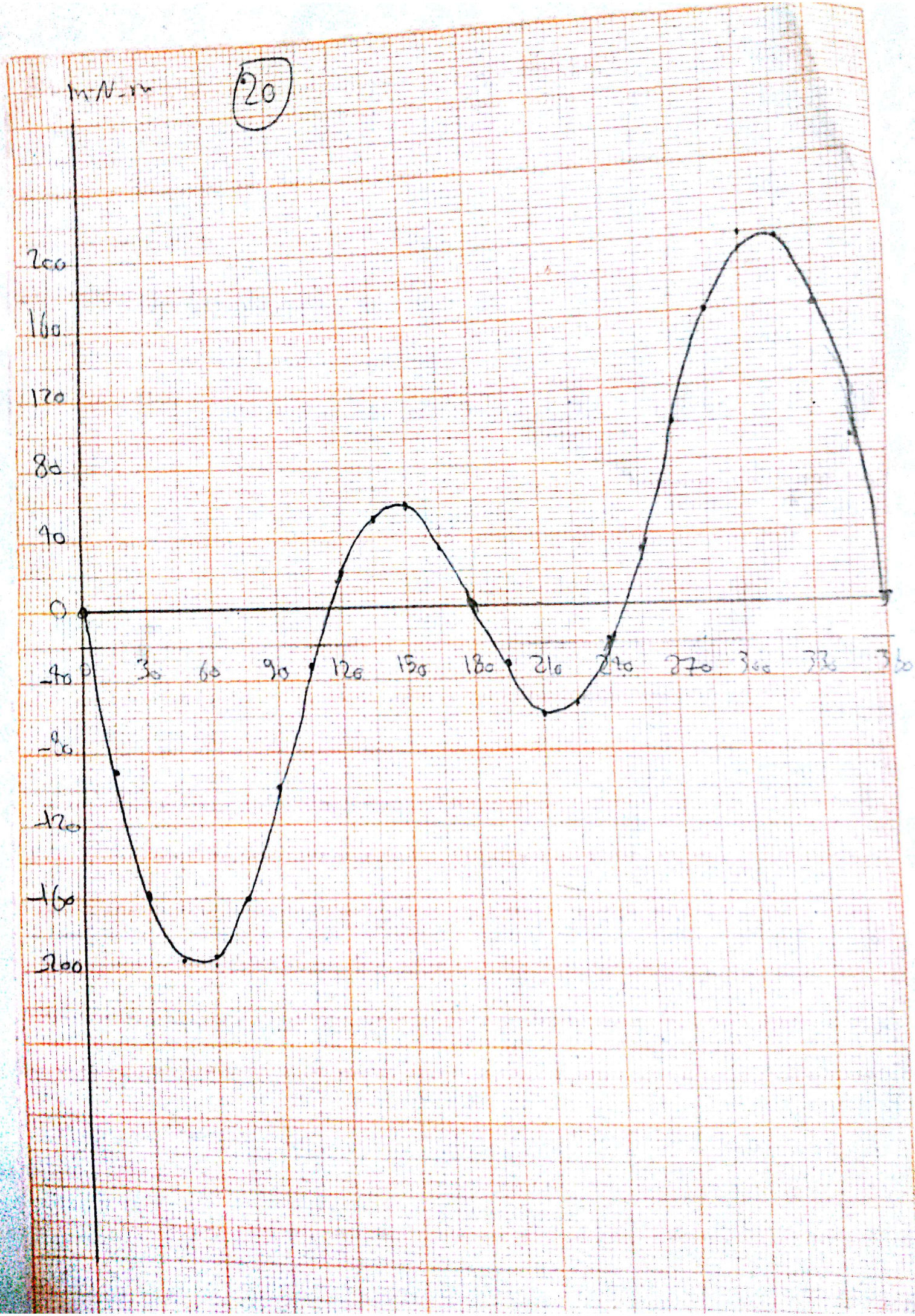

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